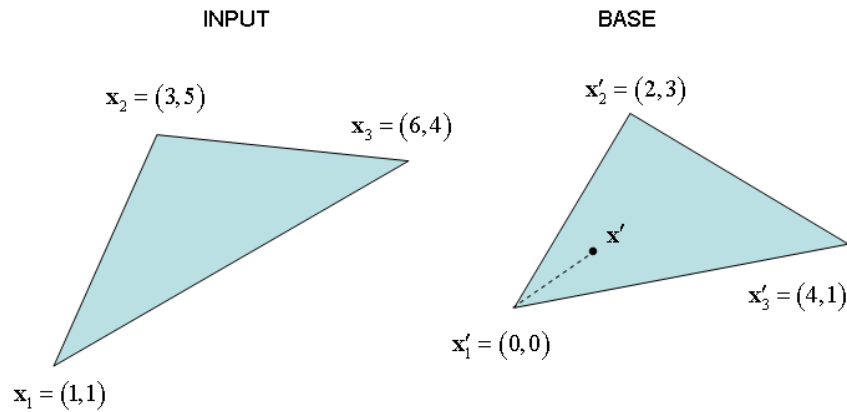


Chapter 7: Geometry  
Questions and Exercises

1. In the context of shape description and analysis in images, briefly describe the utility of *homogenous* coordinates.
2. Write down the individual matrices for the transformations of translation, rotation and scaling in homogeneous coordinates.
3. What is the single matrix that will effect a translation of [10, -10], an overall scaling by a factor of 2 and a rotation by 30 degrees ?  
Use Matlab to apply this sequence of transformations to the unit square (defined by the coordinates of its 4 corners) whose centre is initially located at coordinates [-10, +10]. Plot the initial and transformed shapes on a single plot (you may find the *patch* function useful to produce a solid colour representation of the shape).
4. What do you understand by the geometric transformation known as warping ?
5. Describe the difference between a global affine transformation and a piecewise affine transformation. What are the respective advantages of both approaches ?
6. Consider the problem of mapping the texture from each point inside an input triangle to a corresponding point inside the base triangle depicted in the diagram below.

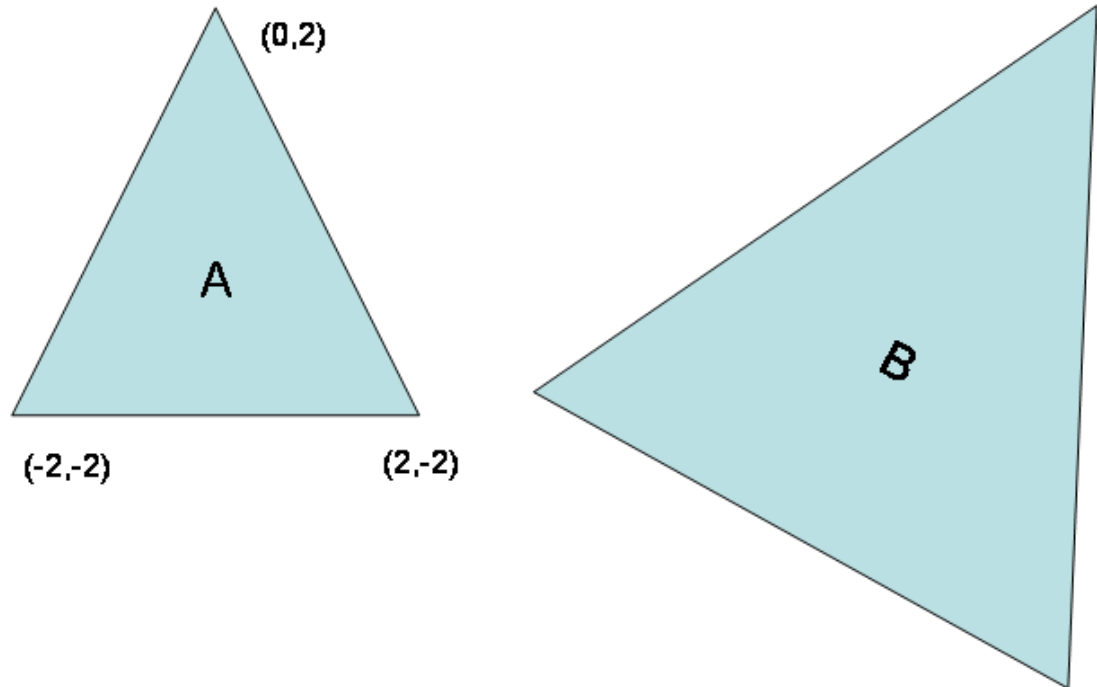


An arbitrary point within the base triangle may be expressed in terms of the triangle vertices in homogeneous matrix form as –

$$\begin{bmatrix} x'_1 & x'_2 & x'_3 \\ y'_1 & y'_2 & y'_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \quad \text{i.e. } \mathbf{T}\underline{\alpha} = \mathbf{x}'$$

where  $\alpha, \beta$  and  $\gamma$  are constants.

- i. Using the explicit coordinates of the vertices indicated in the diagram and the point inside the base triangle  $\mathbf{x}' = (1,1)$ , solve the system of equations for the vector  $\underline{\alpha}$ .
- ii. Calculate the *corresponding point* of the base coordinates (1,1) in the input triangle.



7. Write down a point configuration matrix in *homogeneous* coordinates for the coordinates of the vertices in triangle A.

The triangle is subjected to an affine transformation to produce the object B on the right. This transformation consists of translation in the x and y directions by 10 and -2 units respectively, a scaling by factor 1.5 and a clockwise rotation by angle  $30^\circ$ .

Derive a matrix in *homogeneous* coordinate form which achieves this transformation.

What are the coordinates of the vertices of the transformed triangle B ? Is this transformation shape-preserving ?

8. Explain the steps in the Procrustes alignment procedure, making clear its basic purpose and the role of shape-preserving transformation.

9. Consider the image below (left) (to which illumination compensation has been applied (right). Attempt to produce a similar result to that shown by the following procedure.

Load in the image (lface.png)

Identify a number of points in the image which you estimate would have the same intensity under uniform illumination (same reflectivity). Note the coordinates of the points can be obtained using the Matlab function *ginput*.

Assuming that the recorded image  $i(x, y)$  can be modelled as the product of the illumination function  $s(x, y)$  and the reflectivity  $r(x, y)$  - i.e.

$i(x, y) = s(x, y)r(x, y)$ , perform a least-squares fit of a 2-D plane to these points to estimate the illumination function  $\hat{s}(x, y)$ .

- (Note that the equation of a plane is  $ax + by + cz = d$  which can be written as
- $z = \alpha + \beta x + \gamma y$  where  $z$  represents the intensity as a function of the coordinates  $(x, y)$ . Use the Matlab function *polyfit* to estimate the coefficients of the equation of the plane.)

Then use the Matlab function *polyval* to calculate the estimated illumination function at all points within the  $\hat{s}(x, y)$

Divide the recorded image to estimate the uniformly illuminated image or

reflectivity function  $r(x, y) = \frac{i(x, y)}{\hat{s}(x, y)}$ . The compensated image shows a saturated

(maximum brightness) region in the hair towards the edge of the image. Explain how this is likely to have occurred.

Display the calculated reflectivity using the *imagesc* function. Finally, shift and normalize the data to lie within the normal 8-bit range before displaying using *imshow*.

