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The Matrix form of the imaging equation

Starting from the continuous form of the imaging equation –

$$g(x,y)=\iint f(x',y')h(x-x',y-y')dx'dy'+n(x,y)$$
$$g(x,y)=f(x,y)**h(x,y)+n(x,y) \quad (1)$$

Assuming a sampling of M points at interval $\Delta x'$ in the x' direction and N points at interval $\Delta y'$ in the y' direction and that the output domain is similarly sampled on an M_O by N_O grid. The integral is thus replaced by a summation –

$$g(x,y)=\sum_{i=1}^{M}\sum_{j=1}^{N} f(x'_i,y'_j)h(x,y;x'_i,y')\,\Delta x'\,\Delta y'+n(x,y) \quad (2)$$

with *x* and *y* now *discrete* variables.

If now we simply re-order all the values in the 2-D input domain and all the values in the 2-D output domain into *one-dimensional column vectors* (e.g. according to a simple convention of storing the elements column by column[1]), we can express this as –

$$g(p)=\sum_{q=1}^{MN} f(q)h(p,q)\,\Delta A+n(p) \qquad \text{with } p=1,2...,M_O N_O-1,M_O N_O$$
$$\text{and } q=1,2........,MN-1,MN \quad (3)$$

where the indices *q* and *p* run through all points in the input and output domains respectively. The area element ΔA corresponds to the area of a single pixel in the input domain – for convenience we set this to 1**.** Eq. 3 defines the process of matrix multiplication (i.e. $g_j=\sum_k H_{jk} f_k+n_j$) so eq. 3 that it may be formally written in compact matrix-vector notation as –

$$\mathbf{g}=\mathbf{Hf}+\mathbf{n} \quad (4)$$

[1] E.g. fix the y ordinate and run through all x values, move to next y ordinate and run through all x values and so on. This is sometimes called the ‘stacking operator’.