Line spread function

Consider an idealised, infinitesimally thin line which points along the y axis. This can be represented as $f(x,y)=\delta(x)$. The response of a linear, shift invariant system to such an input is called the line-spread function (LSF) and given by the convolution with the system PSF –

$$l_x(x,y)=\delta(x)**h(x,y)$$
$$=\int_{-\infty}^{\infty}dx'\int_{-\infty}^{\infty}dy'\,h(x-x',y-y')\,\delta(x')=\int_{-\infty}^{\infty}h(x,y')\,dy'=l_x(x)$$

Note that this response although physically extending over two dimensions depends only on the variable x and is thus a one-dimensional function.

Consider now taking the 2-D Fourier transform of $l_x(x)$. By definition, this is –

$$F_T\left[l_x(x)\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}l_x(x)\exp-\left[ik_xx+ik_yy\right]dxdy$$

As the *x* and *y* parts are separable, this may be expressed as the product of two 1-D Fourier transforms -

$$F_T\left[l_x(x)\right]=\int_{-\infty}^{\infty}l_x(x)\exp-(ik_xx)\,dx\int_{-\infty}^{\infty}\exp-\left[ik_yy\right]dy$$
$$=L_x(k_x)\,\delta(k_y)$$

Where $L_x(k_x)$ is the 1-D Fourier transform of $l_x(x)$.

However, we may also invoke the convolution theorem in 2-D which says that –

$$F_T\left[l_x(x)\right]=F_T\left[\delta(x)**h(x,y)\right]=H(k_x,k_y)\,\delta(k_y)=H(k_x,0)\,\delta(k_y)$$

where the last step follows from the observation that the product only has a non-zero value when $k_y=0$.

By equating these two forms for $F_T\left[l_x(x)\right]$, we may therefore conclude that $L_x(k_x)=H(k_x,0)$. In other words, the one-dimensional Fourier transform of the line-spread function (for the line x=0 running along the y axis) is equal to a central slice of the optical transfer function $H(k_x,0)$. A simple repetition of the argument above for a line running along the x axis (y=0) yields $L_y(k_y)=H(0,k_y)$. We will forego the formal proof, but by working in a rotated coordinate system it is possible to derive the general result, namely that the 1-D Fourier transform of the LSF oriented at angle θ with respect to the *x* axis is given by the corresponding slice of the OTF (the 2-D Fourier transform of the PSF) oriented at angle θ with respect to the k_x axis. This is illustrated in figure 1. This relationship is a specific expression of the so-called central or projection slice theorem and finds its best known application in the theory of computed tomography.